# Digital Notch Filter Design and Implementation by means of IIR (Infinite Impulse Response) Filter

## Sumário

# 1 Introduction

This report presents how to design a digital notch filter by means of IIR filter. The IIR coefficients are obtained.

A notch filter is mainly used to extract one harmonic component from a signal containing more than one component. Fig. 1 presents how a notch filter works. The tuned frequency ($f_{nf}$) is the allowed-frequency to pass. All other frequency will be attenuated (filtered).

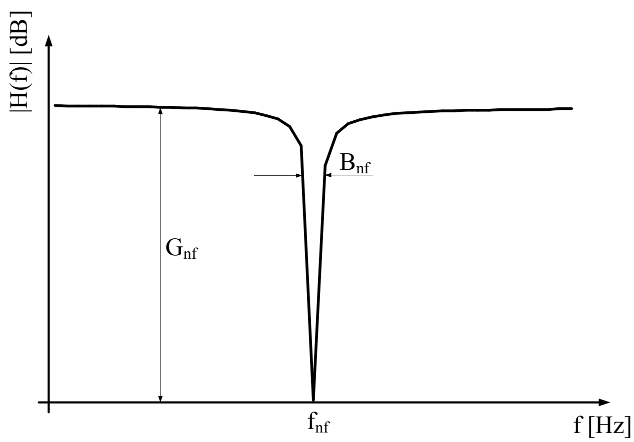


**Figure 1: How a notch filter works.**

Therefore, if you tune the notch filter at 300 Hz, its output signal will be a sinusoidal with 300 Hz. The amplitude will be the same as the input signal. Fig. 2 illustrated better this concept.

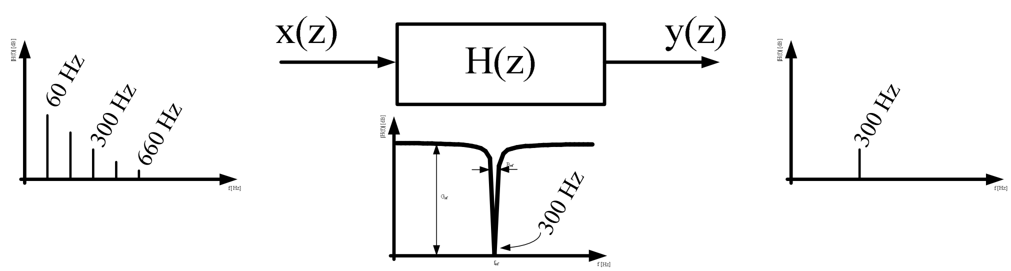


**Figure 2: The concept of how the notch filter works.**

The following section presents a step-by-step way to design a digital notch filter.

# 2 Step 1: Defining the desired parameters

According to Fig. 1, three parameters must be defined in notch filters. They are: the tuned frequency in Hz (fnf), the bandwidth of the notch filter in Hz (Bnf) and the gain in real values (Gnf). The tuned frequency is the component you want to extract from a signal while the bandwidth is the thickness of the signal around the tuned frequency. The value for the bandwidth cannot be null because there is no ideal filter. A good choice is any value around 1% of the tuned frequency. The gain is preferentially chosen as one in order not to lose information from the input signal.

An example will be taken into account for the sake of simplicity. The following parameters are defined in this example.

$$\begin{aligned} f_{nf} &= 300Hz \\ B_{nf} &= 2\pi \cdot 1.59Hz \\ G_{nf} &= 1 \end{aligned} \tag{1}$$

We can say that the notch filter ANGULAR frequency is given by:

$$\omega_{nf} = 2\pi f_{nf} \tag{2}$$

## 3 Step 2: Defining the sampling frequency

Since the notch filter is digital, a sampling frequency (fa) must be defined. It is recommended to set the sampling frequency a value much higher than the tuned frequency. In this example, the sampling frequency is given by:

$$f_a = 30kHz \tag{3}$$

Therefore, the sampling period is given by:

$$T_a = \frac{1}{f_a} \tag{4}$$

## 4 Step 3: The digital notch filter transfer function

The digital notch filter transfer function is given by:

$$H(z) = \frac{y(z)}{x(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}} \tag{5}$$

Or in a recursive way as:

$$y(n) = \left[b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)\right] - \left[a_0 x(n) + a_1 x(n-1) + a_2 x(n-2)\right] \tag{6}$$

Where $n$ is the current sample.

The next step is to obtain the notch filter coefficients based on the parameters given in step 1.

## 5 Step 4: Obtaining the digital notch filter coefficients

The coefficients are obtaining by discretizing an analog notch filter by means of the Z-transformation. The procedure of the Z-transformation will be omitted here.

Let's define a constant called $C_t$ and given by:

$$C_t = \frac{G_{nf} B_{nf}^{\ 2}}{2\sqrt{\omega_{nf}^{\ 2} - 0.25 B_{nf}^{\ 2}}} e^{-0.5 B_{nf} T_a} \sin\left(T_a \sqrt{\omega_{nf}^{\ 2} - 0.25 B_{nf}^{\ 2}}\right) \tag{7}$$

The $b_0$ coefficient is given by:

$$b_0 = G_{nf} B_{nf} T_a \tag{8}$$

The $b_1$ coefficient is given by:

$$b_1 = T_a \left[(-1) G_{nf} B_{nf} e^{-0.5 B_{nf} T_a} \cos\left(T_a \sqrt{\omega_{nf}^{\ 2} - 0.25 B_{nf}^{\ 2}}\right) - C_t\right] \tag{9}$$

The $b_2$ coefficient is null:

$$b_2 = 0.0000 \tag{10}$$

The $a_0$ coefficient is one:

$$a_0 = 1.00000 \tag{11}$$

The $a_1$ coefficient is given by:

$$a_1 = (-1)2e^{-0.5B_{nf}T_a}\cos\left(T_a\sqrt{\omega_{nf}^{2} - 0.25B_{nf}^{2}}\right) \tag{12}$$

The $a_2$ coefficient is given by:

$$a_2 = e^{-B_{nf}T_a} \tag{13}$$

It is recommended to use 12 decimal places.

## 6 Experimental results

A notch filter tuned at 300 Hz was experimentally tested on the DSP TMS320F28335. The input signal is a measured current from a nonlinear load.

The Fig. 3 presents experimental results for a digital notch filter tuned at 300 Hz. On the left: input signal and its harmonic spectra. On the right, the output signal and its spectra. It is evident that the notch filter extracted the 300 Hz component from the input signal. The amplitude at the 300Hz component in the input and output are different because the process to visualize a digital variable in the oscilloscope. A DAC (digital-to-analog) converter had to be used and the amplitude had to be adjusted in order to fit its limitation.

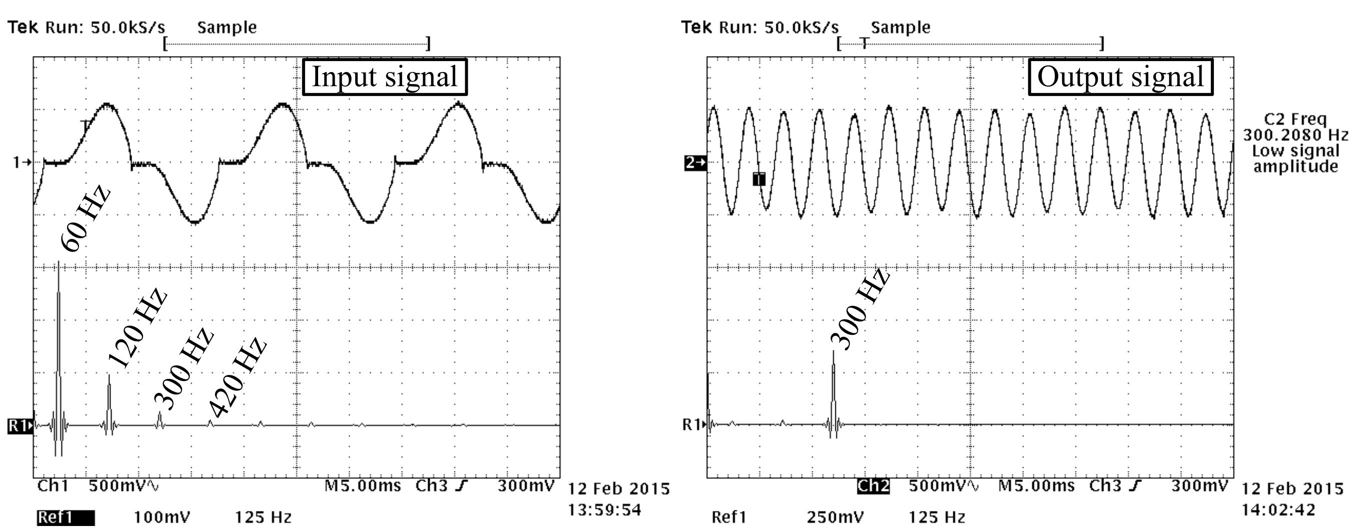


**Figure 3: Experimental results for a digital notch filter tuned at 300 Hz. On the left: input signal and its harmonic spectra. On the right, the output signal and its harmonic spectra.**