The Conservative Power Theory – Current Decomposition in Single and Three-Phase System

Content

1 Preface

This report presents the Conservative Power Theory (CPT) and its simulation circuits. The simulation is freely available on https://sites.google.com/site/busarellosmartgrid/home.

2 Introduction

The CPT was first introduced by Tenti *et al* and it was recently reformulated [1]. The CPT is a time-domain theory that can be applied to single- or multiphase system with or without neutral conductor. It is based on the orthogonal decomposition of electrical variables, resulting electrical quantities with physical meaning. Each quantity represents electrical characteristics like consumed active power, stored energy, phase displacement between voltage and current, unbalancing and so on. Moreover, the CPT does not need any variable transformation. Some applications of the CPT are presented in [2]–[7].

3 Formulas for single-phase

The CPT definitions, valid for any periodic waveform, are:

The average value of a variable x is given by (1).

$$\overline{x} = \frac{1}{T}\int_0^T x(t)dt \tag{1}$$

The time-integral of a variable x is given by (2).

$$x_{\int}(t) = \int_0^T x(\tau)d\tau \tag{2}$$

The unbiased integral of a variable x is given by (3).

$$\hat{x}(t) = x_{\int}(t) - \overline{x}_{\int}(t) \tag{3}$$

where the second term of (3) is the average value of (2). The unbiased term means average value.

The time-derivative of a variable x is given by (4).

$$\breve{x}(t) = \frac{dx(t)}{dt} \tag{4}$$

The unbiased integral and the time-derivative present the following properties:

$$\begin{aligned}
&\hat{\breve{x}} = \breve{\hat{x}} = x \\
&\langle x, \breve{x}\rangle = \langle x, \hat{x}\rangle = 0 \\
&\langle x, \breve{y}\rangle = -\langle \breve{x}, y\rangle \\
&\langle x, \hat{y}\rangle = -\langle \hat{x}, y\rangle \\
&\langle \breve{x}, \hat{y}\rangle = \langle \hat{x}, \breve{y}\rangle = -\langle x, y\rangle
\end{aligned} \tag{5}$$

where $\langle \cdot,\cdot \rangle$ is the internal product.

The RMS value of a variable x is represented by (6).

$$X_{rms} = \|x\| \tag{6}$$

For a given voltage *v(t)* and a current *i(t)*, the active current, responsible for carrying active power, is given by (7).

$$i_a(t)=\frac{\langle v(t),i(t)\rangle}{\|v\|^2}v(t) \tag{7}$$

In a similar way, the reactive current, responsible for carrying reactive energy, is given by (8).

$$i_r(t)=\frac{\langle \hat{v}(t),i(t)\rangle}{\|v\|^2}\hat{v}(t) \tag{8}$$

The residual current is given by (9).

$$i_v(t)=i(t)-i_a(t)-i_r(t) \tag{9}$$

The residual current contains all quantities that do not carry neither active nor reactive power.

4 Formulas for three-phase (with unbalance)

The three-phase active current is given by (10).

$$i_{a(a,b,c)}=\frac{\langle v_{(a,b,c)},i_{(a,b,c)}\rangle}{\|v_{(a,b,c)}\|^2}v_{(a,b,c)} \tag{10}$$

The three-phase reactive current is given by (11).

$$i_{r(a,b,c)}=\frac{\langle \hat{v}_{(a,b,c)},i_{(a,b,c)}\rangle}{\|v_{(a,b,c)}\|^2}\hat{v}_{(a,b,c)} \tag{11}$$

The three-phase void current is given by (12).

$$i_{v(a,b,c)}=i_{(a,b,c)}-i_{a(a,b,c)}-i_{r(a,b,c)} \tag{12}$$

The three-phase active balanced current is given by (13).

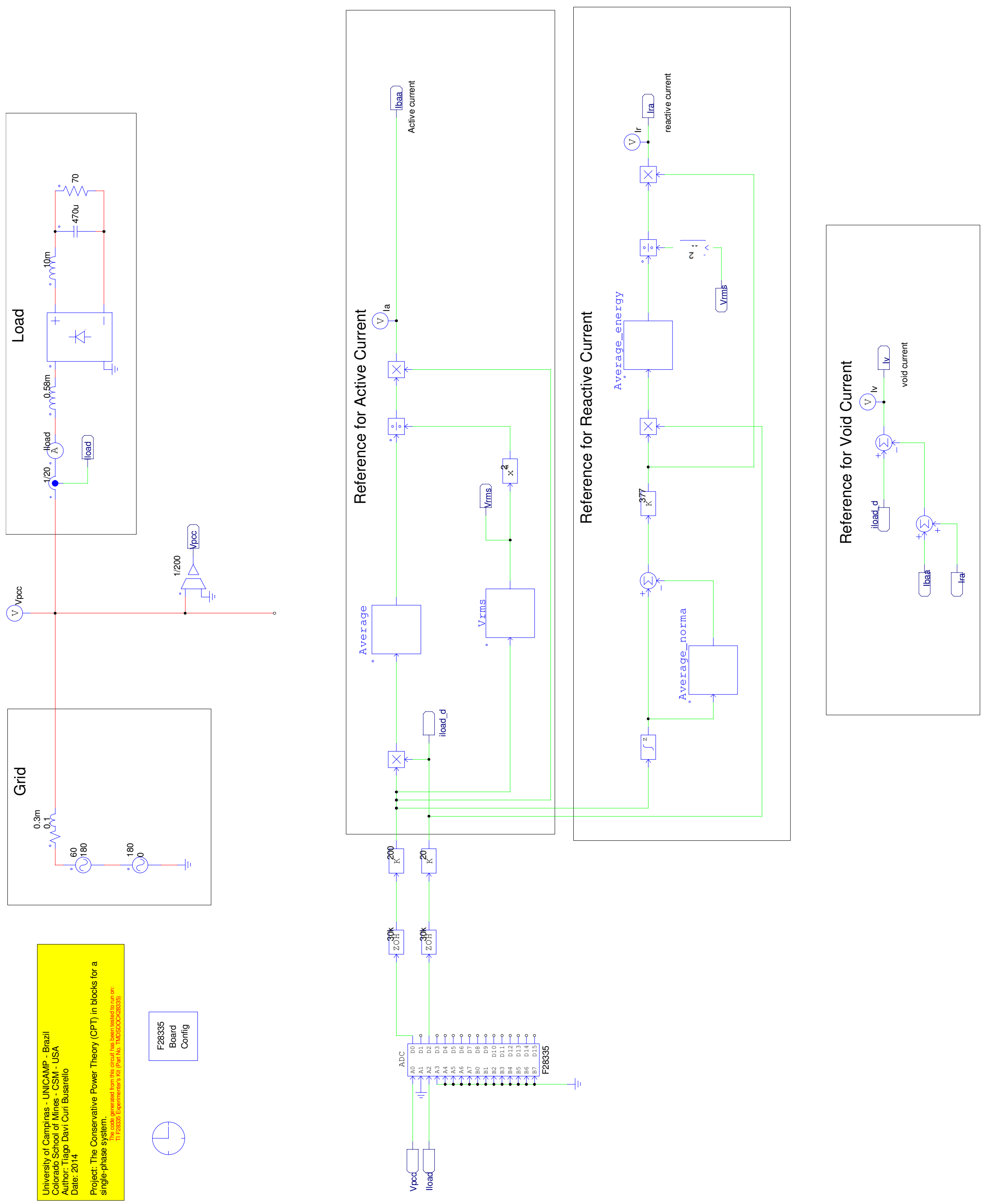
$$i_{abal(a,b,c)}=\frac{\langle v_a i_a\rangle+\langle v_b i_b\rangle+\langle v_c i_c\rangle}{\|v_a\|^2+\|v_b\|^2+\|v_c\|^2}v_{(a,b,c)} \tag{13}$$

The three-phase reactive balanced current is given by (14).

$$i_{rbal(a,b,c)}=\frac{\langle \hat{v}_a i_a\rangle+\langle \hat{v}_b i_b\rangle+\langle \hat{v}_c i_c\rangle}{\|\hat{v}_a\|^2+\|\hat{v}_b\|^2+\|\hat{v}_c\|^2}\hat{v}_{(a,b,c)} \tag{14}$$

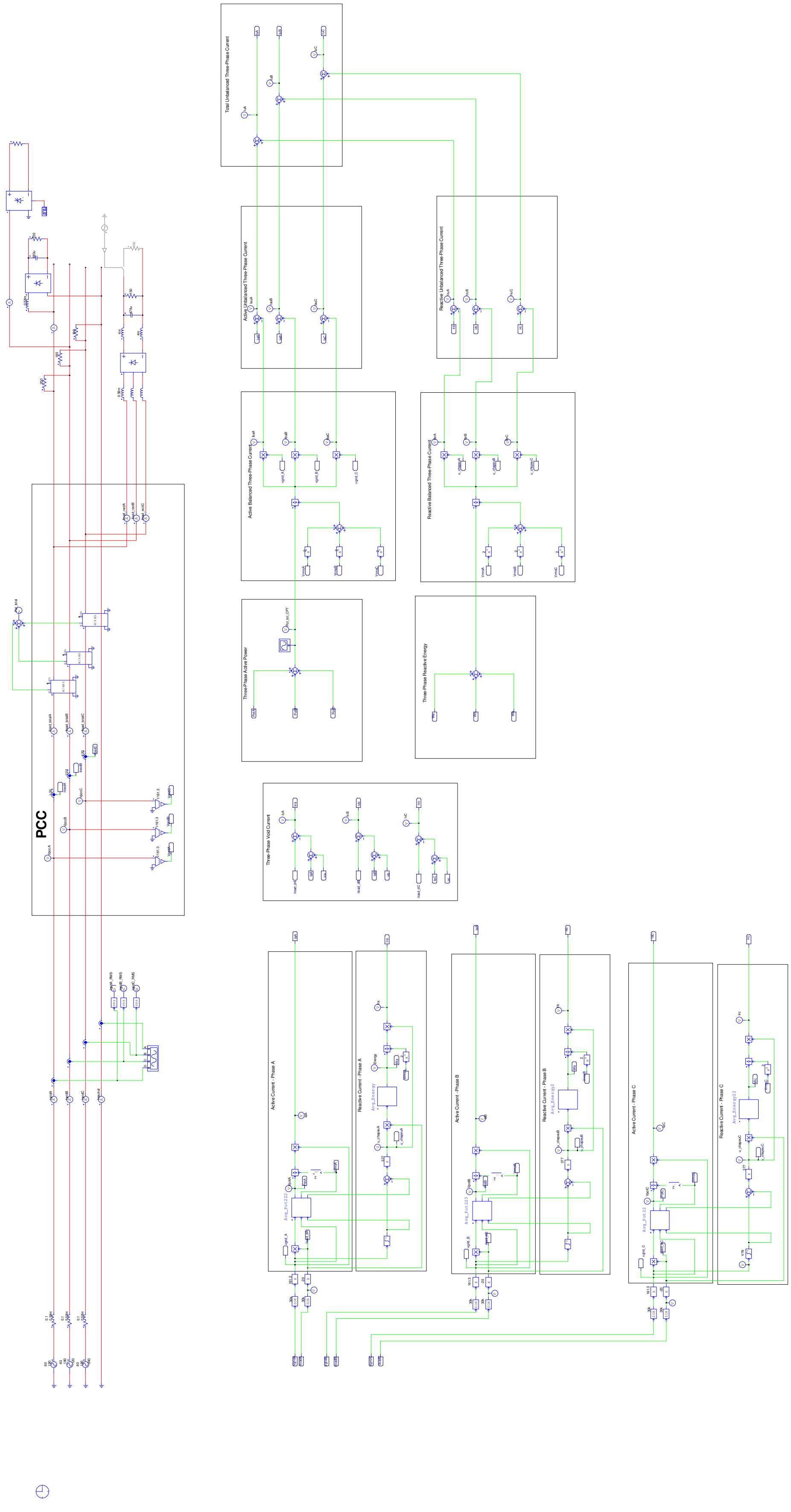
5 Simulation for single-phase

High-quality figure. Make a zoom



6 Simulation for three-phase

High-quality figure. Make a zoom



7 References

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